Based on K. H. Rosen: Discrete Mathematics and its Applications.

Lecture 20: Permutations and Combinations. Section 6.3

1 Permutations and Combinations

1.1 Permutations

A **permutation** of a set of distinct objects is an ordered arrangement of these objects. We also are interested in ordered arrangements of some of the elements of a set. An ordered arrangement of r elements of a set is called an r-permutation.

Example 1. In how many ways can we select three students from a group of five students to stand in line for a picture? In how many ways can we arrange all five of these students in a line for a picture?
Ans: First, note that the order in which we select the students matters. There are five ways to select the first student to stand at the start of the line. Once this student has been selected, there are four ways to select the second student in the line. After the first and second students have been selected, there are three ways to select the third student in the line. By the product rule, there are $5 \cdot 4 \cdot 3 = 60$ ways to select three students from a group of five students to stand in line for a picture.
To arrange all five students in a line for a picture, we select the first student in five ways, the second in four ways, the third in three ways, the fourth in two ways, and the fifth in one way. Consequently, there are $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways to arrange all five students in a line for a picture.

Theorem 2. *If n is a positive integer and r is an integer in the range $0 \leq r \leq n$, then there are*

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1)$$

r-permutations of a set with n distinct elements.

Corollary 3. *If n and r are integers with $0 \leq r \leq n$, then*

$$P(n,r) = \frac{n!}{(n-r)!}.$$

1.2 Combinations

We now turn our attention to counting unordered selections of objects.

Example 4. How many different committees of three students can be formed from a group of four students.
Ans: To answer this question, we need only find the number of subsets with three elements from the set containing the four students. We see that there are four such

subsets, one for each of the four students, because choosing three students is the same as choosing one of the four students to leave out of the group. This means that there are four ways to choose the three students for the committee, where the order in which these students are chosen does not matter.

Definition 5. An r-combination of elements of a set is an unordered selection of r elements from the set. Thus, an r-combination is simply a subset of the set with r elements.

Theorem 6. *The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer in the range $0 \leq r \leq n$, equals*

$$C(n, r) = \frac{n!}{r!(n-r)!}.$$